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Study of heat transfer from buried nuclear waste canisters

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INTRODUCTION

It has been proposed that radioactive waste from nuclear power plants be disposed of in cylindrical containers by burying them under the surface of the earth. To carry out safety analysis and to gauge the impact of this proposal on the environment it is necessary to determine flow patterns and heat transfer rates in the vicinity of these containers. The present work gives a summary of analytical and numerical results for temperature distribution in and around a canister buried in a saturated porous medium. Heating of the canister surface takes place because of the decay of radioactive waste contained within it. It is important to know the maximum and minimum temperatures on the cylinder surface since they decide the magnitude of the transport coefficients and the extent of the thermal stresses. Heat transfer from the cylinder to its surroundings will occur due to one of the following mechanisms: conduction, buoyancy-driven convection of the pore fluid and forced convection due to natural ground water movement. Solutions for these problems are available when the surface of the cylinder has a prescribed temperature. Results have been presented here for a single and an array of cylinders with specified heat flux on their surface.

FORMULATION

Fluid flow in a saturated homogeneous isotropic porous medium is taken to be governed by Darcy's law,

$$\mathbf{u} = -K(\nabla p + \rho g \mathbf{k})/\mu \tag{1}$$

the incompressibility constraint $\nabla \cdot \mathbf{u} = 0$ and the energy transport equation,

$$T_t + \mathbf{u} \cdot \nabla T = \alpha \nabla^2 T. \tag{2}$$

In the absence of buoyancy effects ρ is a constant and $\nabla \cdot \mathbf{u} = \nabla \cdot K \nabla p = 0$. For flow past a single cylinder buried in a uniform medium, K is a constant and $\nabla^2 p = 0$. Using potential theory the velocity components can be determined as $u - iv = U.\ (1 - R^2/z^2)$ where $z = x + iy$ and $i = \sqrt{-1}$. For an array of canisters we solve the equation $\nabla \cdot K \nabla p = 0$ numerically by assigning a small value for K over the cylinders and unit value in the flow region. Equation (2) has been solved here subject to the constant heat flux condition, $-T_r\ (r = R) = q$. The results are presented in dimensionless form using R as the length scale, R^2/α as the time scale, the approach velocity U as the velocity scale and qR as the temperature scale. In free convection problems the velocity scale is α/R. Convection problems are assumed to have reached steady state since they occur in boundary layers.

RESULTS

Conduction limit

In the absence of a super-imposed flow the conduction problem follows the dimensionless equation,

$$T_t = \nabla^2 T$$

subject to $T(t = 0) = 0$. This equation can be solved by Fourier transforms. For an isothermal boundary condition $T(r = 1) = 1$ the solution for the wall heat flux is

$$-T_r|_{r=1} = \left(\frac{2}{\pi}\right)^2 \int_0^\infty \frac{1}{\beta N(\beta)} \exp(-\beta^2 t)\, d\beta$$

where $N(\beta) = J_0^2(\beta) + Y_0^2(\beta)$. This integral is evaluated numerically by Simpson's rule. The conduction solution also describes the local heat flux for steady forced flow parallel to the axis of a cylinder with t replaced by z/Pe. The latter problem has been solved in [1] using boundary-layer analysis. The two solutions are compared in Table 1.

For a heat flux boundary condition $(-T_r(r = 1) = 1)$ we solve for the wall temperature as,

$$T(1, t) = \int_0^\infty \frac{R^2(\beta)}{\beta N(\beta)} [1 - \exp(-\beta^2 t)]\, d\beta$$

where $N(\beta) = J_1^2(\beta) + Y_1^2(\beta)$ and $R(\beta) = J_0(\beta) Y_1(\beta) - J_1(\beta) Y_0(\beta)$. The value of T attained by an isolated canister can increase further if more canisters are present in its neighbourhood. Consider a symmetric array of five canisters, four of which are placed on a square edge d and the fifth is placed at the centre. The temperature of the central canister is obtained by the principle of linear superposition. Calculations show that the minimum temperature is within 98% of the maximum temperature in Table 2.

Free convection [2]

The boundary-layer form of equations governing buoyancy-driven flow and heat transfer are given below.

Table 1. Comparison of heat flux values on an isothermal cylinder

t, z/Pe	Present	[1]
0.5	2.081	2.06
1	1.649	1.597
5	1.071	1.107

NOMENCLATURE

J, Y	Bessel functions of first and second kind	ΔT	characteristic temperature difference
K	absolute permeability	u, v	velocity components.
p	pressure		
Pe	Peclet number, UR/α	Greek symbol	
Ra	Rayleigh number, $g\beta\Delta TKR/v\alpha$	α	thermal dispersion coefficient.

Table 2. Maximum temperature on the central canister of an array

t	1	10	100	1000	10000
$d = 6$	0.834	3.07	7.98	13.53	19.34
$d = 8$	0.797	2.38	6.81	12.38	18.18
$d = \infty$	0.796	1.64	2.71	3.85	5.02

Vertical cylinder

$$u = Ra\, T; \quad (uTr)_z + (vTr)_r = (rT_r)_r \tag{3}$$

Horizontal cylinders

$$(ru)_r = rT_r\, Ra \sin\theta$$

$$(uT)_\theta + (vrT)_r = (rT_r)_r. \tag{4}$$

In equations (3) amd (4) the notation ($)_r$ stands for partial differentiation with respect to r and Ra is the Rayleigh number. These equations are solved by an integral method that uses a quadratic profile for T with the boundary-layer thickness δ as a parameter. For an isothermal cylinder this profile is $T = \{1-[(r-1)/\delta]\}^2$ and for the constant heat flux problem $T = (\delta/2)\{1-[(r-1)/\delta]\}^2$. δ is determined by integrating equations (3) and (4) from $r = 1$ to $r = 1+\delta$. The Nusselt number is defined in the present work as $-T_r$ ($r = 1$) for the isothermal problem and $1/T$ ($r = 1$) for the constant heat flux problem. In each case it can be shown to be $2/\delta$. Solving equations (3) and (4) using the profiles given above yields the following.

Isothermal:

Vertical

$$\delta^3/9 + \delta^2/2 = 10z/Ra$$

Horizontal

$$\int_1^{1+\delta} F\,\mathrm{d}r = \int_0^\theta 2\,\mathrm{d}\theta/\delta$$

where

$$F = \frac{2\,Ra\sin\theta}{\delta^2}\left[1-\left(\frac{r-1}{\delta}\right)\right]^2\left(\frac{(1+\delta)^3}{r} + \frac{r^2}{3} - \frac{r}{2}(1+\delta)\right).$$

Constant heat flux:

Vertical

$$6\delta^3 + \delta^4 = 120z/Ra$$

Horizontal

$$\int_1^{1+\delta} F\,\mathrm{d}r = \theta$$

where

$$F = \frac{Ra\sin\theta}{2r\delta}\left(\frac{r^3-(1+\delta)^3}{3}\right) - \frac{1}{2}\left(\frac{1+\delta}{\delta}\right)[r^2-(1+\delta)^2][r-(1+\delta)]^2.$$

For the isothermal problem the solutions given above are compared with [1] and [3] in terms of the average Nusselt number, see Table 3.

Original results are given in Table 4 for the constant heat flux problem. The vertical cylinder here has a height of ten units.

Forced convection

The problem of cross-flow past a heated cylinder is now considered. The boundary-layer form of equation (2) is $(-2\sin\theta T)_\theta = (1/Pe)\ (rT_r)_r$ which can be integrated between $r = 1$ and $1+\delta$ using a quadratic profile for T. This yields, the following

Isothermal

$$\phi' + 2\cot\theta\phi = -6\operatorname{cosec}\theta/Pe$$

Constant flux

$$\phi' + \cot\theta\phi = -3\operatorname{cosec}\theta/Pe$$

where $\phi = \delta^2$, $\phi' = \mathrm{d}\phi/\mathrm{d}\theta$ and the initial condition is $\phi(\pi) = 3/Pe$ since by symmetry ϕ' is zero here. The equations for ϕ are integrated using a fourth-order Runge–Kutta method. Results for the isothermal problem are compared below in Table 5 with those given in [4].

The boundary-layer method (BLM) is not valid at low Peclet numbers; it is also not useful in studying interaction effects between canisters. The complete steady state solution of equation (2) has been obtained here using a Galerkin finite element method (FEM). Details of this procedure is described in the author's work reported elsewhere [5].

Table 3. Average Nusselt number for an isothermal cylinder

z/Ra	0.001	0.01	0.1	1.0	10.0	Remarks
Nu (present)	14.35	8.64	2.86	1.005	0.384	Vertical
Nu [1]	14.12	9.0	2.96	1.046	0.427	
Ra	10	100	1000	2500	10000	Remarks
Nu (present)	1.496	4.35	13.21	20.72	41.13	Horizontal
Nu [3]	1.263	3.99	12.63	19.97	39.95	

Table 4. Average Nusselt number for a cylinder with a prescribed heat flux

Ra	1	10	100	1000	5000
Vertical	0.586	1.183	2.45	5.16	8.76
Horizontal	0.689	1.39	2.89	6.07	10.26

Table 5. Average *Nu* for cross-flow past an isothermal cylinder

Pe	0.1	1	5	10	50	100
Present	0.232	0.734	1.64	2.32	5.19	7.34
[4]	0.221	0.697	1.56	2.20	4.93	6.97

Table 6. Average *Nu* for cross-flow past a cylinder with a constant heat flux

Pe	0.01	0.1	1	10	50	100
FEM	0.525	0.53	0.837	2.46	5.42	7.18
BLM	0.0836	0.265	0.836	2.65	5.91	8.36

Table 7. Average *Nu* on the central canister of an array with a constant heat flux

Pe	$d = 6$	8	∞
0.1	0.247	0.354	0.525
1	0.478	0.596	0.837
10	2.56	2.52	2.46

Table 6 shows a comparison between the two methods for the constant flux problem. The agreement between them is good at $Pe > 1$ though at $Pe > 100$ the accuracy of FEM on a prescribed grid drops.

The FEM code has been used to determine interaction effects among five canisters placed in a symmetric array as described earlier. The average Nusselt number on the central canister is given in Table 7.

At low Peclet numbers *Nu* on the central canister decreases, i.e. the wall temperature increases due to the presence of the neighbouring canisters. At higher values of *Pe* the cylinders are effectively isolated. In an array the local velocity around the central canister can, however, be larger than for a single canister because of the area blocked. This can lead to further increase in the average Nusselt number. This increase is visible in Table 7.

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